Solution to Assignment 4

Supplementary Problems

1. Let D be a region lying on the first quadrant and Ω the solid obtained by rotating D around the y-axis. Show that the volume of Ω, $|\Omega|$, is given by the formula

$$|\Omega| = |D|d \ ,$$

where $|D|$ is the area of D and d is the distance traveled by the centroid of D. Suggestion: Back to the Riemann sums.

Solution. Let P be a partition on D. Using standard notations, the Riemann sum $\sum_{j,k} 2\pi x_j^* \Delta x_j \Delta y_k$ (where (x_j^*, y_k^*) is a tag point in R_{jk}) approximates the volume of Ω. Taking limits, we have

$$
\begin{aligned}
|\Omega| &= \iint_D 2\pi x \, dA \\
&= 2\pi |D| \frac{1}{|D|} \iint_D x \, dA \\
&= 2\pi \overline{x} |D| \ .
\end{aligned}
$$

The term $d = 2\pi \overline{x}$ is the distance traveled by the centroid. This result is called Theorem of Pappus.